Eur Respir J 1997; 10: 685–689
DOI: 10.1183/09031936.97.10030685
Printed in UK - all rights reserved

European Respiratory Journal
ISSN 0903 - 1936

Estimation of random errors in respiratory resistance and reactance measured by the forced oscillation technique

R. Farré, M. Rotger, D. Navajas

Estimation of random errors in respiratory resistance and reactance measured by the forced oscillation technique. R. Farré, M. Rotger, D. Navajas. ©ERS Journals Ltd 1997.

ABSTRACT: The forced oscillation technique (FOT) allows the measurement of respiratory resistance (*R*rs) and reactance (*X*rs) and their associated coherence (γ^2). To avoid unreliable data, it is usual to reject *R*rs and *X*rs measurements with a γ^2 <0.95. This procedure makes it difficult to obtain acceptable data at the lowest frequencies of interest. The aim of this study was to derive expressions to compute the random error of *R*rs and *X*rs from γ^2 and the number (N) of data blocks involved in a FOT measurement.

To this end, we developed theoretical equations for the variances and covariances of the pressure and flow auto- and cross-spectra used to compute *R*rs and *X*rs.

Random errors of *R*rs and *X*rs were found to depend on the values of *R*rs and *X*rs, and to be proportional to $((1-\gamma^2)/(2\cdot N\cdot\gamma^2))^{1/2}$. Reliable *R*rs and *X*rs data can be obtained in measurements with low γ^2 by enlarging the data recording (*i.e.* N).

Therefore, the error equations derived may be useful to extend the frequency band of the forced oscillation technique to frequencies lower than usual, characterized by low coherence.

Eur Respir J 1997; 10: 685–689.

Lab. Biofisica i Bioenginyeria, Facultat de Medicina, Universitat de Barcelona, Spain.

Correspondence: R. Farré
Lab. Biofisica i Bioenginyeria
Facultat de Medicina
Casanovas 143
E-08036 Barcelona
Spain

Keywords: Coherence
forced oscillation
impedance errors
respiratory resistance

Received: February 26 1996
Accepted after revision November 30 1996

This work was supported, in part, by Comisión Interministerial de Ciencia y Tecnología (CICYT; SAF96-0076).

The forced oscillation technique (FOT) is a noninvasive method which allows assessment of respiratory resistance (*R*rs) and reactance (*X*rs) during spontaneous breathing. This technique has been used increasingly in recent years, and has been shown to be potentially useful in different applications, such as bronchoconstriction and bronchodilation tests [1–4], epidemiological studies [5], anaesthesia and intensive care [6–8].

Most of the methodological aspects concerning the FOT have already been studied, and technical recommendations have recently been made [9]. However, one of the issues that needs more clarification concerns the criteria for assessing data reliability in FOT measurements [9]. Indeed, the only expressions available [10–12] to estimate the random errors in FOT data correspond to the modulus and phase of impedance and not to the conventional representation in terms of *R*rs and *X*rs. In the absence of a direct method to quantify the errors of *R*rs and *X*rs, it is a common practice to use a conservative criterion to reject measured data which are presumably affected by a non-negligible error. The most widespread criterion to warrant reliable *R*rs and *X*rs measurements is to use the coherence function (γ^2) [10] to set a threshold for the acceptance of the data: usually *R*rs and *X*rs with associated γ^2 <0.95 are rejected. Such a procedure provides data that are reasonably free from error [13]. Nevertheless, it suffers from the disadvantage of making it difficult, or even impossible, to collect reliable *R*rs and *X*rs data at frequencies with considerable interest from a physiopathological viewpoint (below ≈4 Hz in healthy adults, and below ≈8 Hz in patients or in children).

The aim of this work was to derive equations that would allow us to compute the random errors in measured *R*rs and *X*rs from the associated γ^2. These equations could be useful in the FOT, since the conventional γ^2 threshold criterion may be avoided, thereby making it possible to obtain reliable estimates of *R*rs and *X*rs in measurements with low coherence. This could extend the frequency range of FOT to frequencies lower than usual.

Errors of resistance and reactance

In most FOT applications, *R*rs and *X*rs are computed from wide-band random or pseudorandom signals processed by the cross-spectra method [9]. This data-processing is widely used in system analysis [11], and was first implemented in the field of respiratory mechanics by Michaelson *et al.* [10] and Landser *et al.* [13]. *R*rs and *X*rs are computed from the auto- and cross-spectra of the recorded pressure and flow signals, which are estimated by averaging the Fourier transforms of a number (N) of independent data blocks. This method also makes it possible to compute the associated coherence, γ^2, which is an indirect index of the signal-to-noise ratio in the signals recorded at the different frequencies.

To deduce the expressions for the relative errors of resistance (ε(*R*rs)) and reactance (ε(*X*rs)), we derived equations for the random errors of the real and imaginary parts of a general transfer function measured by the spectra analysis (Appendix). Taking into account that respiratory impedance (*Z*rs = *R*rs + j·*X*rs; $j^2 = -1$)) is a

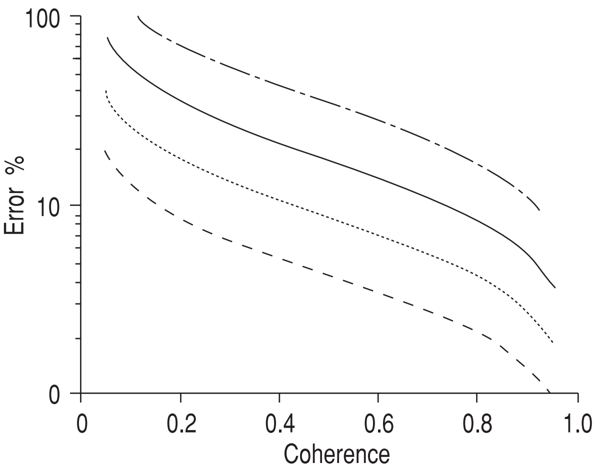

Fig. 1. – Error of R_{rs} and X_{rs} normalized to the modulus of impedance, Equation (4), as a function of coherence for different numbers of data blocks (N). – • – : N=4; —— : N=16; - - - - : N=64; – – –: N=256. R_{rs}: resistance of the respiratory system; X_{rs}: reactance of the respiratory system.

mechanical transfer function of the respiratory system relating flow and pressure, it follows from the Appendix that the errors of the real (R_{rs}) and imaginary (X_{rs}) parts of Z_{rs} are:

$$\varepsilon(R_{rs}) = ((1-\gamma^2)/(\gamma^2\cdot 2\cdot N))^{1/2}\cdot(1 +(X_{rs}/R_{rs})^2)^{1/2} \quad (1)$$

$$\varepsilon(X_{rs}) = ((1\ \gamma^2)/(\gamma^2\cdot 2\cdot N))^{1/2}\cdot(1+(R_{rs}/X_{rs})^2)^{1/2} \quad (2)$$

By multiplying Equation (1) and (2) by R_{rs} and X_{rs}, respectively, and after algebraic rearrangements, the absolute errors of resistance (SD(R_{rs})) and of reactance (SD(X_{rs})) were found to depend on the modulus of impedance $|Z_{rs}|$ ($|Z_{rs}| = (R_{rs}^2 + X_{rs}^2)^{1/2}$) according to:

$$SD(R_{rs}) = SD(X_{rs}) = |Z_{rs}|\cdot((1-\gamma^2)/(\gamma^2\cdot 2\cdot N))^{1/2} \quad (3)$$

Therefore, the absolute errors of R_{rs} and X_{rs} normalized to the modulus of impedance depend only on N and γ^2:

$$SD(R_{rs})/|Z_{rs}| = SD(X_{rs})/|Z_{rs}| = ((1-\gamma^2)/(\gamma^2\cdot 2\cdot N))^{1/2} \quad (4)$$

as shown in figure 1, where these errors are plotted as a function of coherence for different lengths of the signal (*i.e.* N).

Discussion

In this work, we derived equations allowing us to estimate the random errors of measured R_{rs} and X_{rs} by applying the general theory of spectral analysis in the FOT. In contrast to the approaches followed in other works [13–15], where the analysis of errors in R_{rs} and X_{rs} included both random and bias errors, we focused attention on the random errors, on the assumption that bias is minimized by means of the different procedures proposed in the literature to correct for the main potential sources of bias error (poor frequency response of transducers [16], shunt of the extrathoracic upper airways [17], and correlated noises due to breathing [18, 19]). It is noteworthy that, in our analysis, we implicitly assumed the hypotheses of stationarity and linearity of the respiratory system, as is usual in the field of the FOT. Consequently, in the particular applications of FOT where nonlinearities [20, 21] or nonstationarities [8] play an important role, the effective R_{rs}, X_{rs}, γ^2 and their derived error equations must be interpreted, carefully, taking into account the characteristics of the system and the measuring conditions [22]. However, the respiratory system seems to be reasonably linear and stationary [23, 24] in the conventional applications of FOT to assess R_{rs} and X_{rs} in spontaneously breathing subjects [9].

Random error in a given FOT measurement cannot be minimized, since it appears as the result of computing R_{rs} and X_{rs} from a limited length of signals (*i.e.* N), which are affected by a given level of measurement noise. The equations derived in this work are intended to estimate the errors, $\varepsilon(R_{rs})$ and $\varepsilon(X_{rs})$, from the R_{rs}, X_{rs} and γ^2 values computed from the total number (N) of data blocks collected, which are the best possible estimates of R_{rs}, X_{rs} and γ^2. Errors of R_{rs} and X_{rs} could also be assessed from the SD of N' different estimates of R_{rs} and X_{rs}, each one computed by averaging N/N' data blocks from the same pressure and flow data. Nevertheless, this procedure is inadequate from the point of view of the cross-spectra analysis. Indeed, the cross-spectra method is based on the elimination of the effects of the uncorrelated noises affecting the pressure and flow signals by means of averaging the spectra of a theoretically infinite number of data blocks [10, 11]. Therefore, the method is more efficient in providing reliable estimates of R_{rs} and X_{rs} as N is increased. This fact is illustrated by the most extreme case (N'=N), where partial R_{rs} and X_{rs} estimates are computed by averaging only one data block and, consequently, there is no noise-cancelling effect of the cross-spectra method.

As shown by Equations (1) and (2), random errors of R_{rs} and X_{rs} depend on the total number (N) of data blocks averaged to estimate spectra, which corresponds to the duration of the measurement, and on the value of the coherence (γ^2), which indirectly reflects the signal-to-noise ratio. On the one hand, as γ^2 approaches 1 the random errors are progressively reduced, and when $\gamma^2=1$ the error would disappear (fig. 1). On the other hand, as the number (N) of data blocks averaged increases the random errors also decrease. In particular, figure 1 shows that for the usual coherence threshold ($\gamma^2=0.95$) and number of blocks (N ≈4–16) the errors in R_{rs} and X_{rs} are reasonably low (≈4–8%). Moreover, this figure shows that increasing the length of the measurement (*i.e.* enlarging N) would progressively reduce the random error for any given value of γ^2. Consequently, the use of the derived error equations may allow us to implement a FOT measurement procedure with a modified rationale from the viewpoint of the assessment of data reliability. Indeed, instead of performing a data acquisition of fixed duration, computing R_{rs}, X_{rs} and γ^2 and rejecting data with low coherence as usual, it is possible to set a target error and to proceed with the data acquisition (*i.e.* increasing N) until $\varepsilon(R_{rs})$ and $\varepsilon(X_{rs})$ reach the target error level. As derived from Equation (4), to ensure a given error (SD) in R_{rs} or X_{rs}, it is required that the number of data blocks was $N=(1-\gamma^2)/(2\cdot\gamma^2)\cdot(SD/|Z_{rs}|)^{-2}$. This relationship is plotted in figure 2, showing the length of data (N) required to achieve normalized errors of 5, 10, 15 and 20% as a function of coherence.

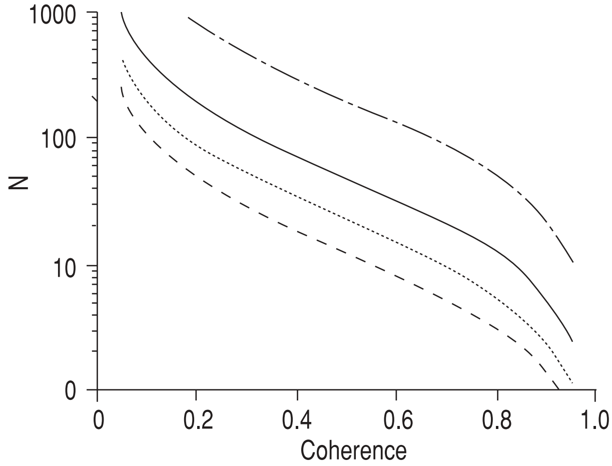

Fig. 2. – Number of data blocks (N) required to reduce normalized errors of R_{rs} and X_{rs} to 5% (– • –), 10% (——), 15% (- - - -) and 20% (– – –) as a function of coherence. R_{rs}: resistance of the respiratory system; X_{rs}: reactance of the respiratory system.

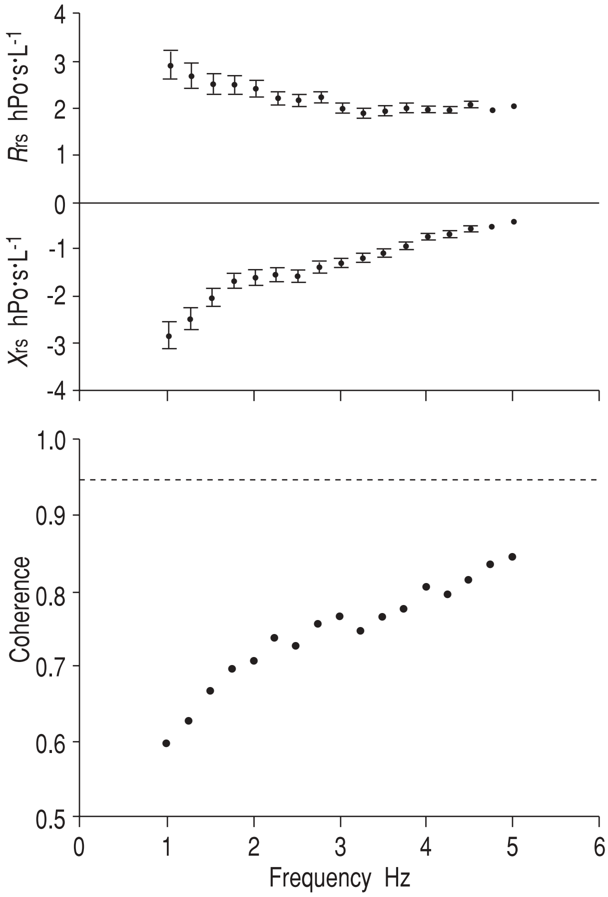

Fig. 3. – R_{rs} and X_{rs} and the errors of R_{rs} and X_{rs} (top panel) estimated from coherence (bottom panel) according to Equation (3). R_{rs} and X_{rs} were computed from N=64 data blocks in a low-frequency FOT measurement in a spontaneously breathing healthy subject. The dashed line (bottom panel) indicates the conventional threshold (γ^2 = 0.95) for accepting data. FOT: forced oscillation technique; R_{rs}: resistance of the respiratory system; X_{rs}: reactance of the respiratory system.

The most direct method of increasing N to reduce errors in R_{rs} and X_{rs} is to enlarge the duration of the FOT measurement. In this regard, it is interesting to note that, for a given total length of pressure and flow data, N could be increased by shortening each data block submitted to spectral analysis. Nevertheless, such a procedure would reduce the frequency resolution in the estimation of spectra. For instance, a total data length of 32 s (4,096 samples at a sampling frequency of 128 Hz) could be divided into N=64 blocks of 0.5 s (64 samples) each, or into N=16 blocks of 2 s (256 samples) each. The frequency resolution in the spectra estimation would be of 2 and 0.5 Hz, respectively. The advantage of reducing errors in R_{rs} and X_{rs} estimation in the first instance due to a large N would be balanced by a reduction in the frequency resolution in R_{rs} and X_{rs}. However, the errors resulting from these two different data processing procedures could also be influenced by the possible different values of coherence found, since γ^2 depends on the signal-to-noise ratio, which may vary depending on the width of the frequency window in spectra estimation [25].

Computing the error in R_{rs} and X_{rs} from N and γ^2 instead of setting a γ^2 threshold for accepting data may be particularly useful in FOT measurements at low frequencies in spontaneously breathing subjects. In this application, γ^2 may fall to rather small values due to a poor signal-to-noise ratio, as shown by a representative example in figure 3. This figure plots R_{rs}, X_{rs}, γ^2 and the errors of R_{rs} and X_{rs} between 1 and 5 Hz obtained in a healthy subject as described in detail previously [26]. It is interesting to note that the errors in R_{rs}, Equation (1), and in X_{rs}, Equation (2) were reasonable even at 1 Hz ($\varepsilon(R_{rs})$ = 9.8% and $\varepsilon(X_{rs})$ = 10.2%), although the associated coherence (γ^2=0.60) was far below the typical threshold (γ^2=0.95) used in FOT measurements. This example illustrates the potential interest of using the errors of R_{rs} and X_{rs} as a criterion for acceptance of FOT data. In this case, which is characterized by low coherences (γ^2 0.60–0.85), recording a reasonable number of data blocks (N=64) made it possible to obtain reliable R_{rs} and X_{rs} (fig. 3). By contrast, the conventional rejection criterion based on a coherence threshold would make it impossible to accept any of the computed R_{rs} and X_{rs} data.

Replacing the coherence threshold criterion with one in which random errors of respiratory resistance and reactance estimates are quantified may extend the application of the forced oscillation technique to frequencies that are lower than usual, provided that bias errors are minimized [26].

Appendix: random errors of the real and imaginary parts of a transfer function.

The transfer function (H) of a linear and stationary system can be computed by means of the cross-spectra method as: $H=G_{xy}/G_{xx}$; where G_{xy} is the cross-spectrum between input and output and G_{xx} is the auto-spectrum of the input. As G_{xy} is a complex magnitude ($G_{xy}=C_{xy}+j\cdot Q_{xy}$) and G_{xx} is a real magnitude, the real (R) and imaginary (I) parts of H can be expressed as:

$$R = C_{xy}/G_{xx} \qquad (A1)$$

$$I = Q_{xy}/G_{xx} \qquad (A2)$$

To compute the error in R, we derive the equation: $C_{xy} = R \cdot G_{xx}$, obtained from the expression for R in Equation (A1):

$$\Delta C_{xy} = R \cdot \Delta G_{xx} + G_{xx} \cdot \Delta R \qquad (A3)$$

By rearranging this equation and taking the squares of both sides of the resulting equation, we obtain:

$$(G_{xx} \cdot \Delta R)^2 = (\Delta C_{xy})^2 + (R^2 \cdot \Delta G_{xx})^2 - 2 \cdot R \cdot \Delta C_{xy} \cdot \Delta G_{xx} \qquad (A4)$$

Computing the expected values (E[·]) in this equation and considering that the variance of a magnitude A (Var(A)) is $Var(A)=E[(\Delta A)^2]$, and that the covariance of variables A and B (Cov(A,B)) is $Cov(A,B)=E[\Delta A \cdot \Delta B)]$, Equation (A4) leads to:

$$G_{xx}^2 \cdot Var(R) = Var(C_{xy}) + R^2 \cdot Var(G_{xx}) - 2 \cdot R \cdot Cov(C_{xy},G_{xx}) \qquad (A5)$$

The variances and covariances of the auto- and cross-spectra in Equation (A5) can be expressed in terms of the spectra values [11]:

$$Var(C_{xy}) = (G_{xx} \cdot G_{yy} + C_{xy}^2 - Q_{xy}^2)/(2 \cdot N) \qquad (A6)$$

$$Var(G_{xx}) = G_{xx}^2/N \qquad (A7)$$

$$Cov(C_{xy},G_{xx}) = C_{xy} \cdot G_{xx}/N \qquad (A8)$$

where N is the number of data blocks involved in estimating spectra. The reduction in the variance of spectra estimates Equations (A6)–(A8), when they are computed by multiplying 50% overlapped data blocks by optimal windows, is negligible [27]. Replacing the variances and the covariance in Equation (A5) by their expressions in Equations (A6)–(A8), and after algebraic rearrangements, Equation (A5) can be rewritten as:

$$N \cdot Var(R)/R^2 = (G_{xx} \cdot G_{yy} - |G_{xy}|^2)/(2 \cdot R^2 \cdot G_{xx}^2) \qquad (A9)$$

By using the expressions of H and coherence (γ^2) in terms of the spectra ($H = G_{xy}/G_{xx}$ and $\gamma^2 = |G_{xy}|^2/(G_{xx} \cdot G_{yy})$) in Equation (A9), we can compute the relative error in the real part ($\varepsilon(R)=(Var(R)/R^2)^{1/2}$) as:

$$\varepsilon(R) = ((1-\gamma^2)/(\gamma^2 \cdot 2 \cdot N))^{1/2} \cdot (1 + (I/R)^2)^{1/2} \qquad (A10)$$

Following a similar procedure from the expression of I in Equation (A2), we can also compute the relative error of the imaginary part of H ($\varepsilon(I)$) as:

$$\varepsilon(I) = ((1-\gamma^2)/(\gamma^2 \cdot 2 \cdot N))^{1/2} \cdot (1 + (R/I)^2)^{1/2} \qquad (A11)$$

The impact of using a coherence estimate instead of the actual unknown value of γ^2 in Equations (A10) and (A11) is small for the typical values of N and γ^2, since the random error in coherence estimates is given [11] by:

$$\varepsilon(\gamma^2) = (1-\gamma^2)/\gamma \cdot (2/N)^{1/2} \qquad (A\ 12)$$

References

1. Van Noord JA, Clement J, Van de Woestijne KP, Demedts M. Total respiratory resistance and reactance as a measurement of response to bronchial challenge with histamine. *Am Rev Respir Dis* 1989; 139: 921–926.
2. Van Noord JA, Smeets J, Clement J, Van de Woestijne KP, Demedts M. Assessment of reversibility of airway obstruction. *Am J Respir Crit Care Med* 1994; 150: 551–554.
3. Marchal F, Mazurek H, Habib M, Duvivier C, Derelle J, Peslin R. Input respiratory impedance to estimate airway hyperreactivity in children: standard method *versus* head generator. *Eur Respir J* 1994; 7: 601–607.
4. Phagoo SB, Watson RA, Silverman M, Pride NB. Comparison of four methods of assessing airflow resistance before and after induced narrowing in normal subjects. *J Appl Physiol* 1995; 79: 518–525.
5. Pairon JC, Iwatsubo Y, Hubert C, *et al.* Measurement of bronchial responsiveness by forced oscillation technique in occupational epidemiology. *Eur Respir J* 1994; 7: 484–489.
6. Navajas D, Farré R, Canet J, Rotger M, Sanchis J. Respiratory input impedance in anesthetised paralysed patients. *J Appl Physiol* 1990; 69: 1372–1379.
7. Van de Woestijne KP. The forced oscillation technique in intubated, mechanically-ventilated patients. *Eur Respir J* 1993; 6: 767–769.
8. Peslin R, Felicio da Silva J, Duvivier C, Chabot F. Respiratory mechanics studied by forced oscillations during artificial ventilation. *Eur Respir J* 1993; 6: 772–784.
9. Van de Woestijne KP, Desager KN, Duiverman EJ, Marchal F. Recommendations for measurement of respiratory input impedance by means of the forced oscillation method. *Eur Respir Rev* 1994; 4 (19): 235–237.
10. Michaelson ED, Grassman ED, Peters WR. Pulmonary mechanics by spectral analysis of forced random noise. *J Clin Invest* 1975; 56: 1210–1230.
11. Bendat JS, Piersol AG. Random data. Analysis and measurements procedures. 2nd Edn. New York, John Wiley & Sons, 1986.
12. Miller TK, Pimmel RL. Standard errors on respiratory mechanical parameters obtained by forced random excitation. *IEEE Trans Biomed Eng* 1983; 30: 826–832.
13. Landser FJ, Nagels J, Demedts M, Billiet L, Van de Woestijne KP. A new method to determine frequency characteristics of the respiratory system. *J Appl Physiol* 1976; 41: 101–106.
14. Delavault E, Saumon G, Georges R. Characterization and validation of forced input method for respiratory impedance measurement. *Respir Physiol* 1980; 40: 119–136.
15. Franken H, Clement J, Van de Woestijne KP. Systematic and random errors in the determination of respiratory impedance by means of the forced oscillation technique: a theoretical study. *IEEE Trans Biomed Eng* 1983; 30: 642–651.
16. Delavault E, Saumon G, Georges R. Identification of transducer defect in respiratory impedance measurements by forced random noise: correction of experimental data. *Respir Physiol* 1980; 40: 107–117.
17. Peslin R, Duvivier C, Didelon J, Gallina C. Respiratory impedance measured with head generator to minimize upper airway shunt. *J Appl Physiol* 1985; 59: 1790–1795.
18. Daroczy B, Hantos Z. An improved forced oscillatory estimation of respiratory impedance. *Int J Biomed Comput* 1982; 13: 221–235.

19. Navajas D, Farré R, Rotger M, Peslin R. A new estimator to minimize the error due to breathing in the measurement of respiratory impedance. *IEEE Trans Biomed Eng* 1988; 35: 1001–1005.
20. Lutchen KR, Yang K, Kaczka DW, Suki B. Optimal ventilator waveforms for estimating low-frequency respiratory impedance. *J Appl Physiol* 1993; 75: 478–488.
21. Hantos Z, Petak F, Adamicza A, Daroczy B, Suki B, Lutchen KR. Optimum ventilator waveform for the estimation of respiratory impedance: an animal study. *Eur Respir Rev* 1994; 4 (19): 191–197.
22. Suki B, Lutchen KR. Pseudorandom signals to estimate apparent transfer and coherence functions of nonlinear systems: applications to respiratory mechanics. *IEEE Trans Biomed Eng* 1992; 39: 1142–1151.
23. Rotger M, Peslin R, Farré R, Duvivier C. Influence of amplitude, phases and frequency content of pseudorandom pressure input on impedance data and their variability. *Eur Respir Rev* 1991; 1 (3): 178–182.
24. Farré R, Peslin R, Rotger M, Navajas D. Human lung impedance from spontaneous breathing frequencies to 32 Hz. *J Appl Physiol* 1994; 76: 1176–1183.
25. Farré R, Rotger M. Filtering the noises due to breathing in respiratory impedance measurements. *Eur Respir Rev* 1991; 1 (3): 196–201.
26. Navajas D, Farré R, Armengol J. Optimizing respiratory impedance measurements at low frequencies in spontaneously breathing subjects. *Eur Respir Rev* 1991; 1 (3): 102–205.
27. Harris FJ. On the use of windows for harmonic analysis with the discrete Fourier transform. *Proc IEEE* 1978; 66(1): 51–83.